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CALCULATION OF ALL BIMAGIC 8x8 SQUARES

by Walter Trump and Francis Gaspalou

ABSTRACT

The calculation of the 8x8 bimagic squares was driven from December 2013 to April 2014. The results are given ; explicit data are available on link # [1].

RESULTS

Number of essentially different 8x8-squares (*):

Semi-bimagic (SM):	1 760 208 713
Bimagic (SQ):	**136 244**
SM that lead to SQ (**):	124 991

Number of unique 8x8-squares:

SQ:	192 × 136 244	=	**26 158 848**
SM:	406 425 600 × 1 760 208 713	=	715 393 882 306 252 800

(*) Beside rotations and reflections there are 192 permutations of rows and columns under which all magic 8x8-squares are invariant (ref. [6]). For SM the number of permutations is $(8!/2)^2$.

(**) By permutation of rows and columns, a given SM can be transformed (or not) into one or several SQ

HISTORICAL DATA

The story of the 8x8 bimagic squares takes place between French and German people.

The first 8x8 bimagic square in the world was found in 1890 by Georges Pfeffermann (ref. [1]). He was born German, in Frankfurt, and later obtained the French citizenship (see biography ref. [2])

During the 1890-1910 years, the people who worked on the bimagic squares were essentially French: Pfeffermann who built the first one, Coccoz or his pseudonym Luet, Savard who built the first semi-bimagic one, Huber, Portier, Rilly, Tarry, etc.

Later we find the General E. Cazalas (still a Frenchman) who wrote a master book in 1934 (ref. [4]).

We have to wait the computers and the 2010's years to see new results on the 8x8 bimagic squares:

- Walter Trump (a German) enumerated in 2011 the associative squares and the pandiagonal complete squares,
- Francis Gaspalou (a Frenchman) enumerated in 2012 the Tarry's squares (Greco-Latin, ref. [7]), and in 2013 the Coccoz's squares (ref. [8]) and the Rilly's squares,
- and today they enumerate together the whole set.

PRINCIPLE OF THE CALCULATION

Walter found the idea for the enumeration. He developed a GB32 program with the bit vector method. He was helped by Francis who developed on his side a C program (with the bit vector method) which validated several parts of the results of Walter. Francis verified also that the known sets of squares were inside the whole set.

The complement to 65 allowed to reduce the number of cases. Therefore we had to calculate about half of all ess. diff. SM. Each SM was inspected in order to construct bimagic diagonals.

The SM were built by the bit vector method where the 8 numbers of each line are described by 8 bits which are set in a 64-bit integer. The sets of entries in two parallel lines of a SM have to be disjoint. Say r1 and r2 are the two 64-bit-integers representing row 1 and row 2 then the following condition has to be fulfilled: r1 AND r2 = 0. Such tests can be done extremely fast by processors.

For searching the ess. diff. SM, the number 64 was put in the first cell A1. We had to consider all the 3 789 possible ordered bimagic series (out of 38 039 ; cf ref.[3],[5]) with the number 64 inside for building the first row and the first column.

The task was finally divided in 4 great cases called A, B, C and D according to conditions on the indexes of the rows and columns with the number 64 and on the similar indexes of the rows and columns with the number 1. Cases A and B had number 1 in the first row. In the cases B and C only one of each complementary pair of SQ was considered.

VERIFICATIONS

As already indicated, several parts of the results of the enumeration program in GB32 were checked by a C program written independently, but also:

- we checked that each complement to 65 was inside the whole set,

- we applied to some subsets all the possible permutations of rows and columns for checking that the squares we obtained were inside the initial subset,

- we checked that several sets of known ess. diff. squares were inside the whole set, specially:
* the 841 associative SQ,
* the 1 836 pandiagonal complete SQ,
* the 1 344 Greco-Latin SQ (Tarry's SQ),
* the 10 317 Coccoz's SQ
* the 2 543 Rilly's SQ,

CONCLUSION

The calculation of the 8x8 bimagic squares was possible because:

- we used the bit vector method which is a tremendously efficient method,
- the set of the SM was calculable with our personal computers, i.e. this set was not too large (Walter estimated as early as 2008 that there were 1.76*10^9 ess. diff. SM).

LINKS

[1] Download the explicit list of all ess. diff. bimagic squares of order 8
http://www.trump.de/bimagic-8.zip

[2] Homepage of Walter Trump
http://www.trump.de/magic-squares

[3] Homepage of Francis Gaspalou
http://www.gaspalou.fr/magic-squares

REFERENCES

[1] Tablettes du Chercheur, 15 Janvier 1891 et 1er Fevrier 1891

[2] Biography of Georges Pfeffermann in
http://www.multimagie.com/fr.htm
click on “Bimagique 8”

[3] Liste des 38 039 suites bimagiques de 8, Achille Rilly 1906 (Médiathèque du Grand Troyes, 10000 Troyes)

[4] Général E. Cazalas, Carrés magiques au degré n, Hermann éd., 1934
(this book gives a very complete bibliography)

[5] Maurice Kraitchik, Mathematical Recreation, 1948

[6] William H. Benson and Oswald Jacoby, New Recreations with Magic Squares, 1976

[7] Francis Gaspalou, how many squares are there, Mr. Tarry? February 10, 2012
http://www.multimagie.com/GaspalouTarry.pdf

[8] Francis Gaspalou, revisit of the method of construction of the first bimagic squares, October 14, 2013
http://www.multimagie.com/GaspalouCoccoz.pdf